Chapter 4.1: Angles and Their Measures

A ray is part of a line that extends in one direction.

An angle is created by two rays having the same endpoint.

α

An angle is measured from the initial side to the terminal side.

θ

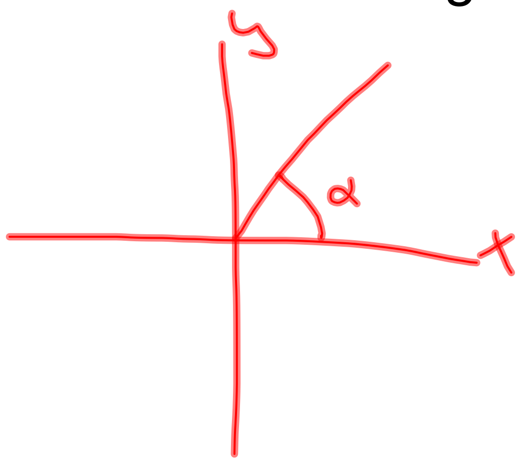
Angles are named using alpha(α), beta (β), gamma(γ), theta(θ)

θ

An angle in standard position -

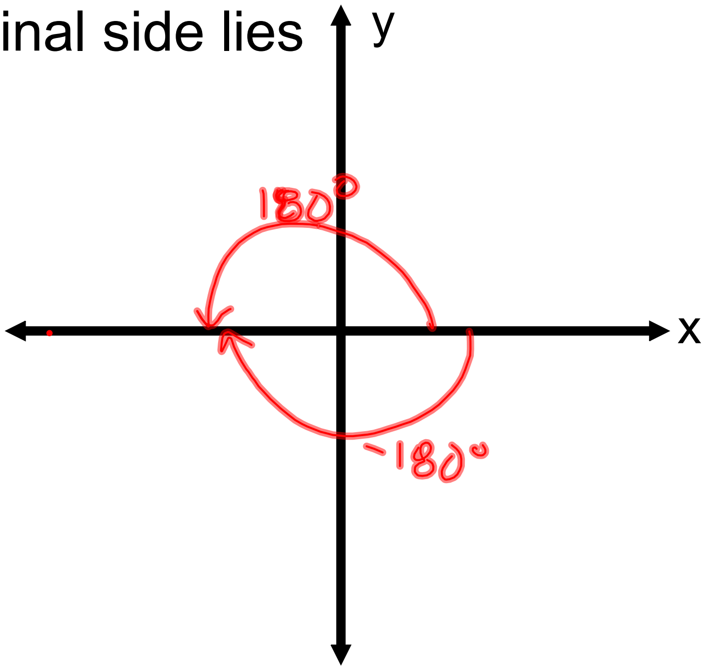
its vertex is at the origin of a rectangular coordinate system

its initial side lies along the positive x-axis



Positive/Negative/Quadrantal Angles

an angle lies in the quadrant in which its terminal side lies



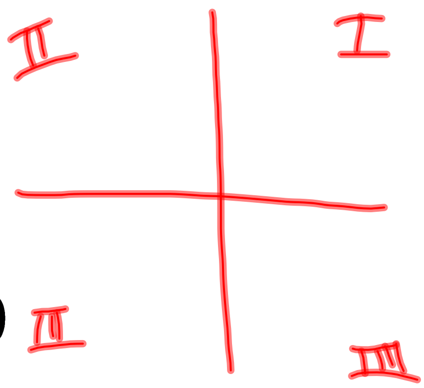
One way to measure angles is in degrees.

Acute $0 < \theta < 90$

Right $\theta = 90$

Obtuse $90 < \theta < 180$

Straight $\theta = 180$



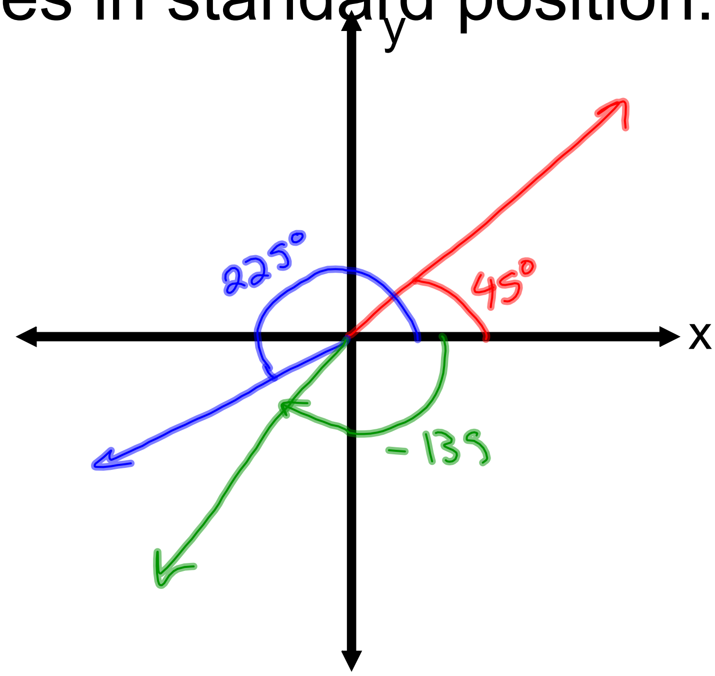
Draw the angles in standard position:

a. $\theta = 45°$

b. $\theta = 225°$

c. $\theta = -135°$

d. $\theta = 405°$



parts of angles are measured in minutes(1' or 1/60 degrees) and seconds(1" or 1/3600 degrees)

$$31°47'12" = 31.787°$$

$$31 + \frac{47}{60} + \frac{12}{3600}$$

Coterminal Angles are angles that share the same terminal position. like 45 and 405

$$x° + k \bullet 360°$$

Find the positive coterminal angle between 0-360

$\theta = 420°$

-360

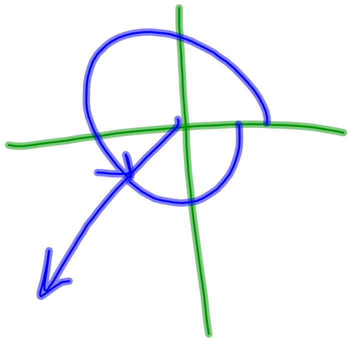
$60°$

$\theta = -120°$

$+360$

$240°$

45°



Finding complements and supplements

Complement angles sum to 90. if x is an angle its complement is 90-x

Supplement angles sum to 180. if x is an angle its supplement is 180-x

Both supplement and complement angles must be positive.

Find the complement and supplement angle, if there is one.

$\theta = 62°$

Comp: 90-62

$28°$

Supp: 180-62

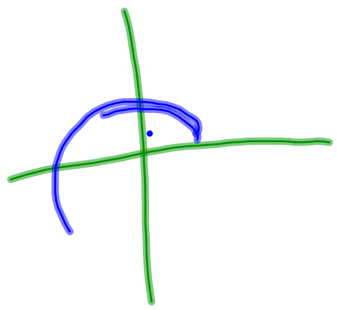
$118°$

$\theta = 123°$

no comp

180-123

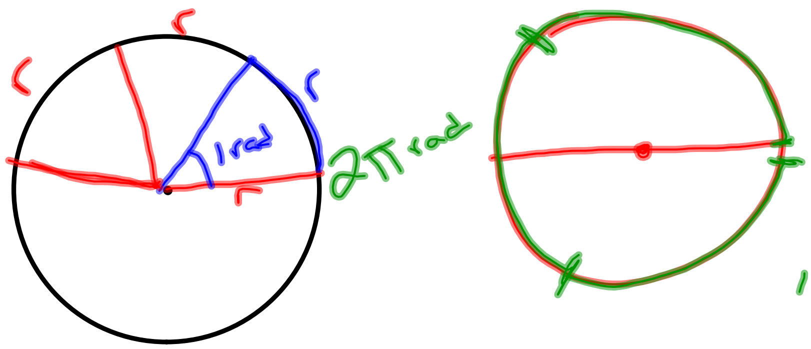
$\boxed{57°}$



Measuring Angles Using Radians:

One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

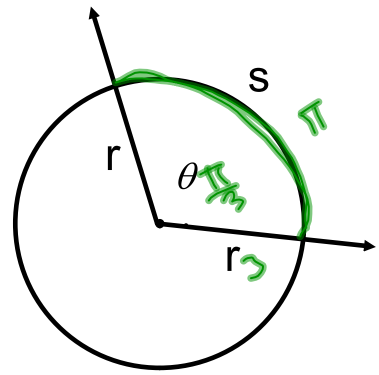
$\frac{\pi}{2}$



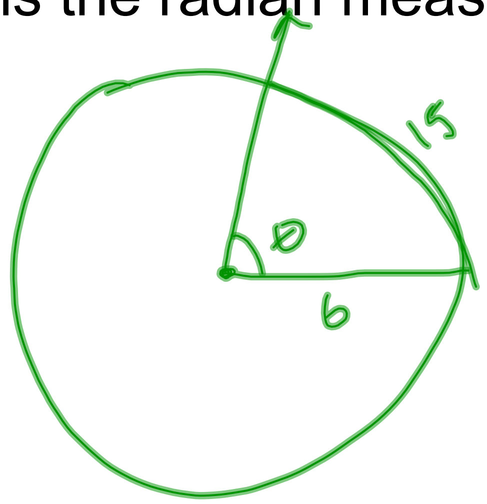
Consider an arc of length s on a circle of radius r. The measure of the central angle, θ, that intercepts the arc is

$$\theta = \frac{s}{r} \text{ radians}$$

$\theta r = s$



A central angle, θ, in a circle of radius 6 inches intercepts an arc of length 15 inches. What is the radian measure of θ?



$\theta = \frac{s}{r}$

$\theta = \frac{15}{6}$

$= \boxed{\frac{5}{2} \text{ rad}}$

Relationship between Degrees and Radians

$$360° = 2\pi$$

$$180° = \pi$$

$\frac{\pi}{6} = 30°$

$$90° = \frac{\pi}{2}$$

D to R: degree $\cdot \frac{\pi}{180}$

R to D: radian $\cdot \frac{180}{\pi}$

Convert the angle to the other measure:

$\theta = 30°$

$\frac{\pi}{6}$

$\theta = 45°$

$\frac{45}{1} \cdot \frac{\pi}{180}$

$\frac{\pi}{4}$

$\theta = -135°$

$-135\left(\frac{\pi}{180}\right)$

$-\frac{3\pi}{4}$

$\theta = \frac{\pi}{3}$ rad

$\frac{\pi}{3} \cdot \frac{180}{\pi}$

$\boxed{60°}$

$\theta = 1$ rad

$\frac{1}{1} \cdot \frac{180}{\pi}$

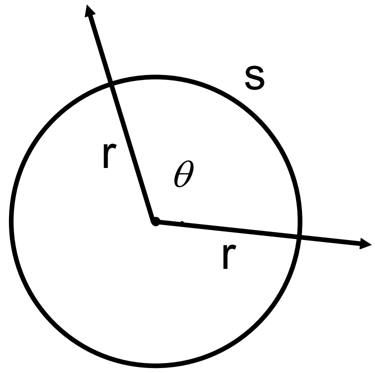
$\boxed{\approx 57.3°}$

$\theta = -\frac{5\pi}{3}$ rad

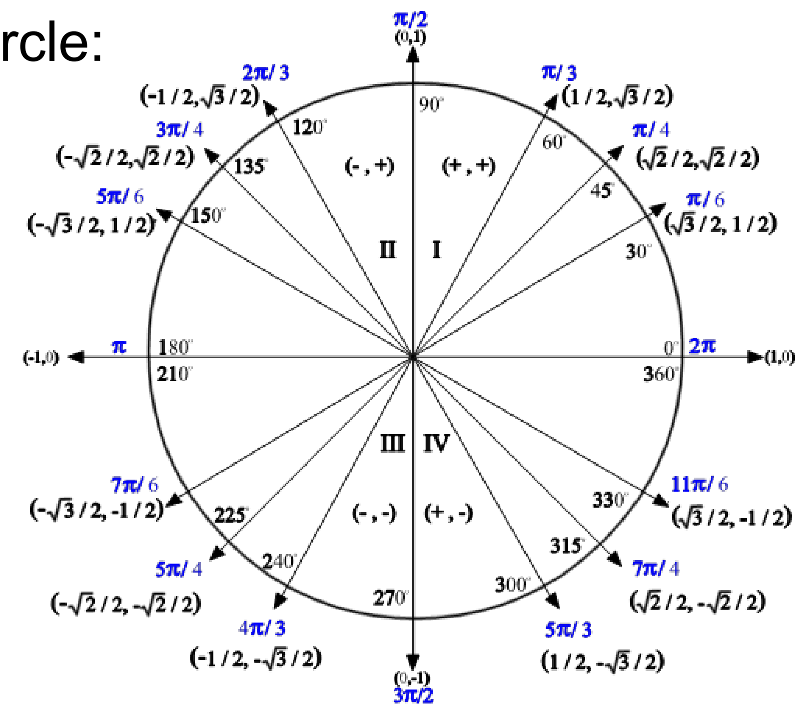
$\frac{-5\pi}{3} \cdot \frac{180}{\pi}$

$\boxed{-300°}$

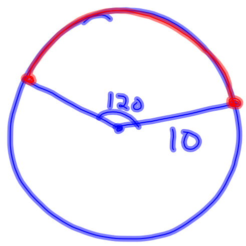
Arc Length: $s = r\theta$



Unit Circle:



A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120.



$(\text{part})(2\pi r)$

$\left(\frac{120}{360}\right)(2\pi(10))$

$\frac{1}{3}(20\pi)$

$\frac{20\pi}{3}$ inches

$s = r\underline{\theta}$, θ must be in radians

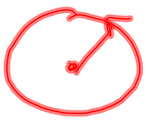
$120^\circ \rightarrow$ radians $\quad \frac{120}{1}\cdot\frac{\pi}{180} = \frac{2\pi}{3}$

$s = \frac{10}{1}\left(\frac{2\pi}{3}\right) = \boxed{\frac{20\pi}{3}\text{ in.}}$

1.5°

Linear and Angular Speed:

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its linear speed is $\boxed{v = \frac{s}{t} = \frac{r\theta}{t}}$ → radians



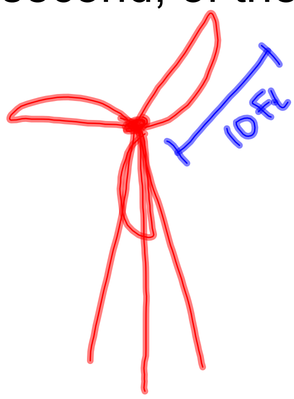
where s is the arc length given by $s = r\theta$, and its angular speed is $w = \frac{\theta}{t}$ $\qquad V = r\frac{\theta}{t}$

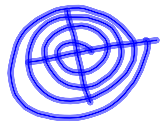
Linear Speed in terms of Angular Speed:

The linear speed, v, of a point a distance r from the center of rotation is given where w is the angular speed in radians per unit of time.

$$v = rw$$

A wind machine used to generate electricity has blades that are 10 feet in length. The propeller is rotating at four revolutions per second. Find the linear speed, in feet per second, of the tips of the blades.





$V = \frac{r\theta}{t}$

$V = \frac{10(8\pi)}{1}$

$= 80\pi$ ft per sec

$\frac{80\pi \text{ ft}}{\text{sec}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}}$

171.4mph

Suggested Problems: Chapter 4.1 pg. 434 #'s 1,3,11,12,19,21,25,29,31,35, 39,43,47,49,55,61,66,69,71,77,81,83